# SIMULINK IMPLEMENTATION OF HODGKIN-HUXLEY SPIKING NEURON MODEL

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# HH Equations - 1

$$C\frac{dV}{dt} = I - \overbrace{\bar{g}_K n^4 \left(V - E_K\right)}^{I_K} - \overbrace{\bar{g}_{Na} m^3 h \left(V - E_{Na}\right)}^{I_{Na}} - \overbrace{g_L \left(V - E_L\right)}^{I_L}$$

$$\frac{dn}{dt} = \frac{n_\infty\left(V\right) - n}{\tau_n\left(V\right)}$$

$$\frac{dm}{dt} = \frac{m_\infty\left(V\right) - m}{\tau_m\left(V\right)}$$

$$\frac{dh}{dt} = \frac{h_\infty\left(V\right) - h}{\tau_h\left(V\right)}$$

Hodgkin, A & Huxley, A. *A Quantitative Description of Membrane Current and its Application to Conduction and Excitation in Nerve.* J. Physiol. 117: 500-544 [1952]

# HH Equations - 2

Where we have:

$$n_\infty(V) = \frac{\alpha_n}{\alpha_n + \beta_n}, \quad m_\infty = \frac{\alpha_m}{\alpha_m + \beta_m}, \quad h_\infty = \frac{\alpha_h}{\alpha_h + \beta_h}$$

$$\tau_n(V) = \frac{1}{\alpha_n + \beta_n}, \quad \tau_m(V) = \frac{1}{\alpha_m + \beta_m}, \quad \tau_h = \frac{1}{\alpha_h + \beta_h}$$

and the parameters $\{\alpha_n, \beta_n, \alpha_m, \beta_m, \alpha_h, \beta_h\}$ correspond to a membrane potential shift and are defined as follows:

$$\alpha_n(V) = 0.01 \frac{10 - V}{\exp\left(\frac{10-V}{10}\right) - 1}, \quad \alpha_m(V) = 0.1 \frac{25-V}{\exp\left(\frac{25-V}{10}\right) - 1}, \quad \alpha_h(V) = 0.07 \exp\left(\frac{-V}{20}\right)$$

$$\beta_n(V) = 0.125 \exp\left(\frac{-V}{80}\right), \quad \beta_m(V) = 4 \exp\left(\frac{-V}{18}\right), \quad \beta_h(V) = \frac{1}{\exp\left(\frac{30-V}{10}\right) + 1}$$

Hodgkin, A & Huxley, A. *A Quantitative Description of Membrane Current and its Application to Conduction and Excitation in Nerve.* J. Physiol. 117: 500-544 [1952]

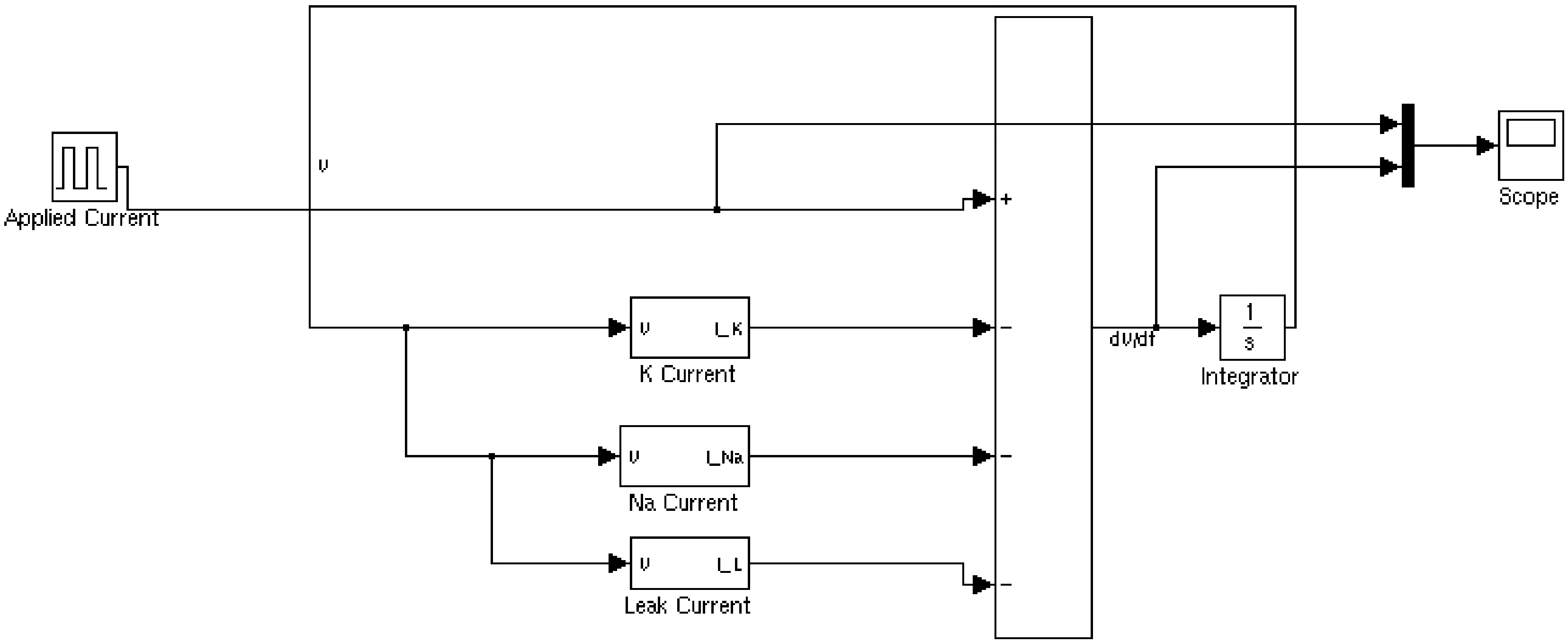
# HH Constants

Furthermore, the shifted nernst equilibrium potentials are

$$E_K = -12\,mV, \quad E_{Na} = 120\,mV, \quad E_L = 10.6\,mV.$$

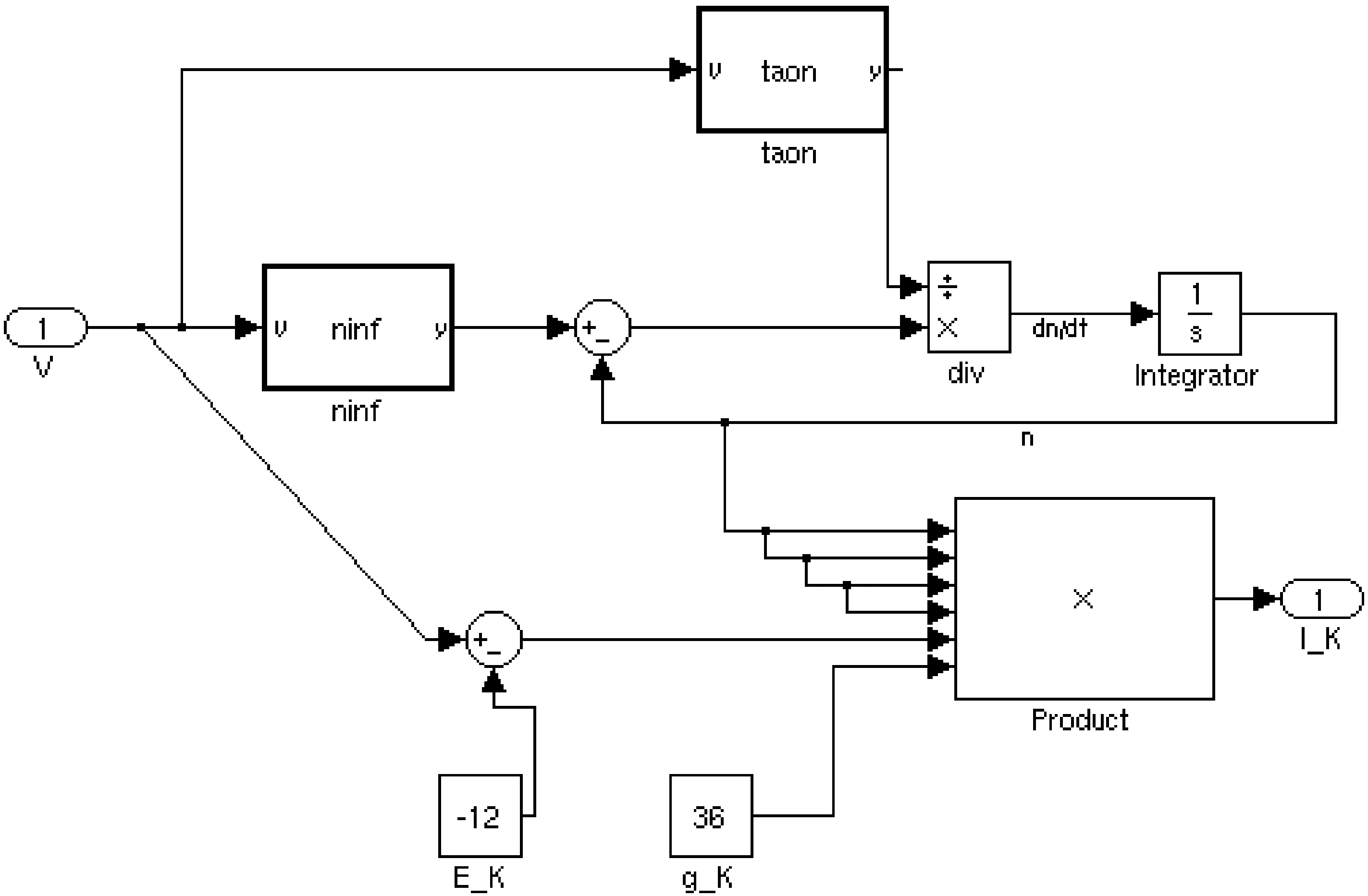
Hodgkin, A & Huxley, A. *A Quantitative Description of Membrane Current and its Application to Conduction and Excitation in Nerve.* J. Physiol. 117: 500-544 [1952]

# HH Main (Level 2)



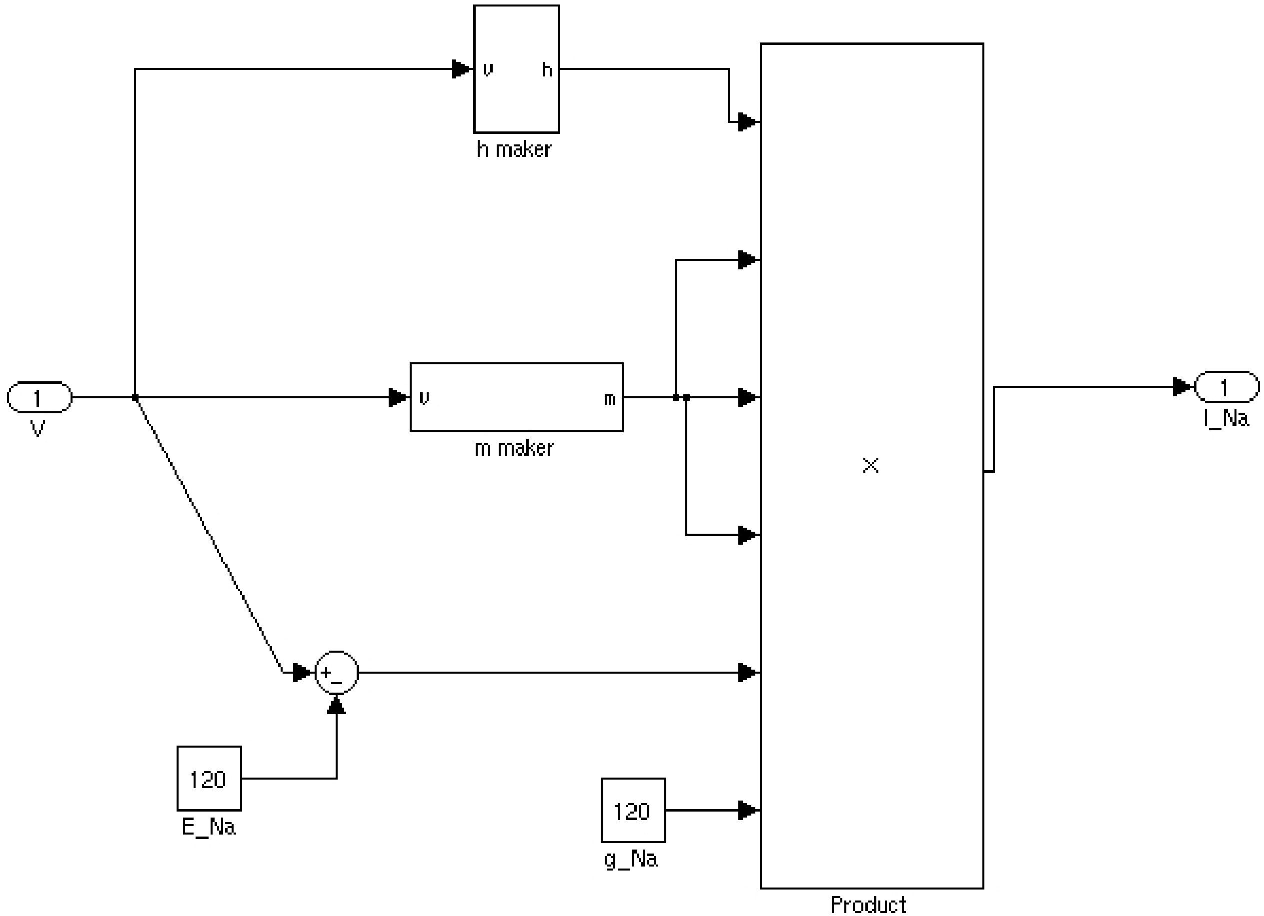

Simulink Block Diagram of HH Neuron (Level 2)

# I_K (Level 1)



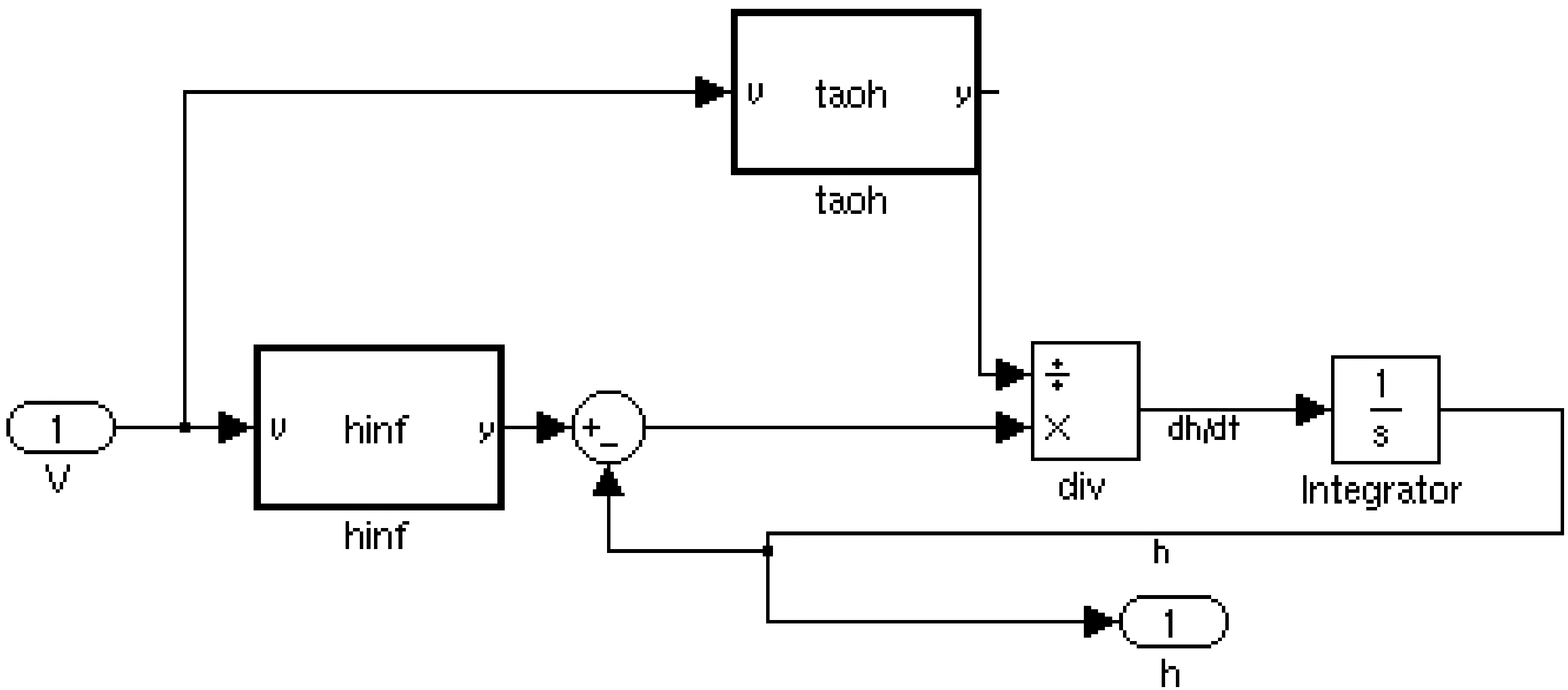

Simulink Block Diagram of I_K (Level 1)
Initial Condition: n=0.317676914060697

# I_Na (Level 1)



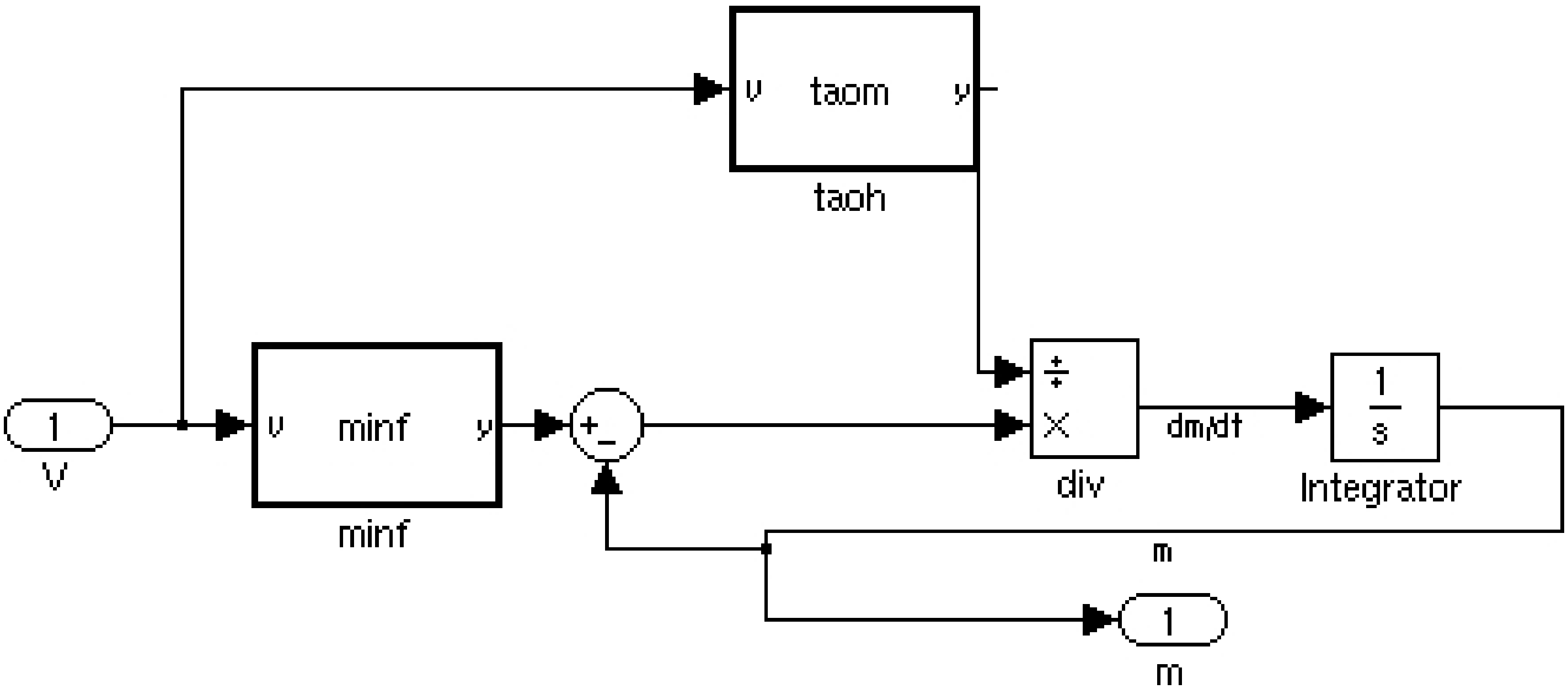

Simulink Block Diagram of I_Na (Level 1)

# I_Na h producer (Level 0)



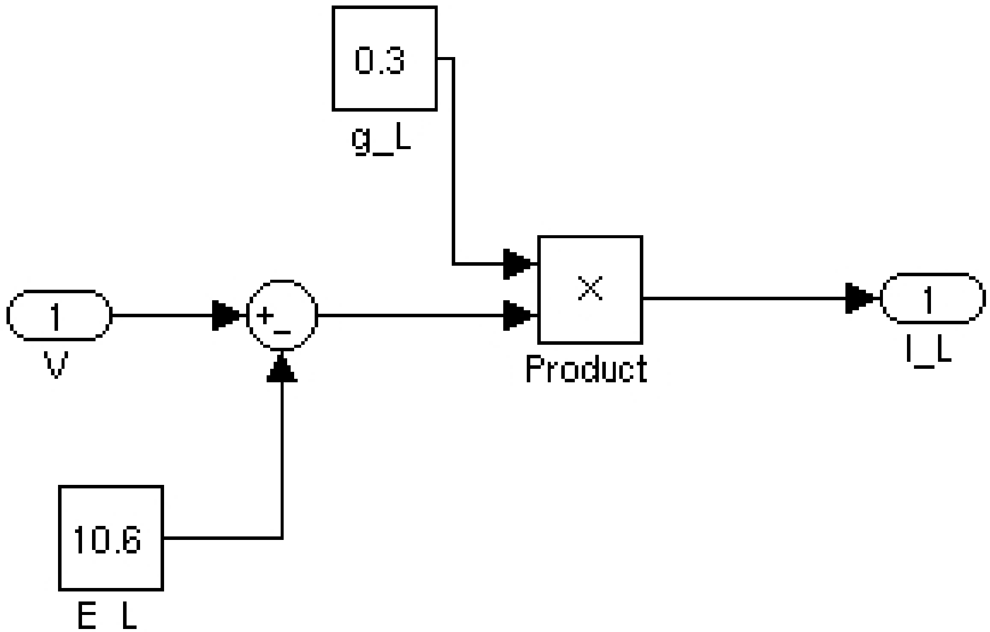

Simulink Block Diagram of I_Na h producer (Level 0)
Initial Condition: h=0.596120753508460

# I_Na m producer (Level 0)



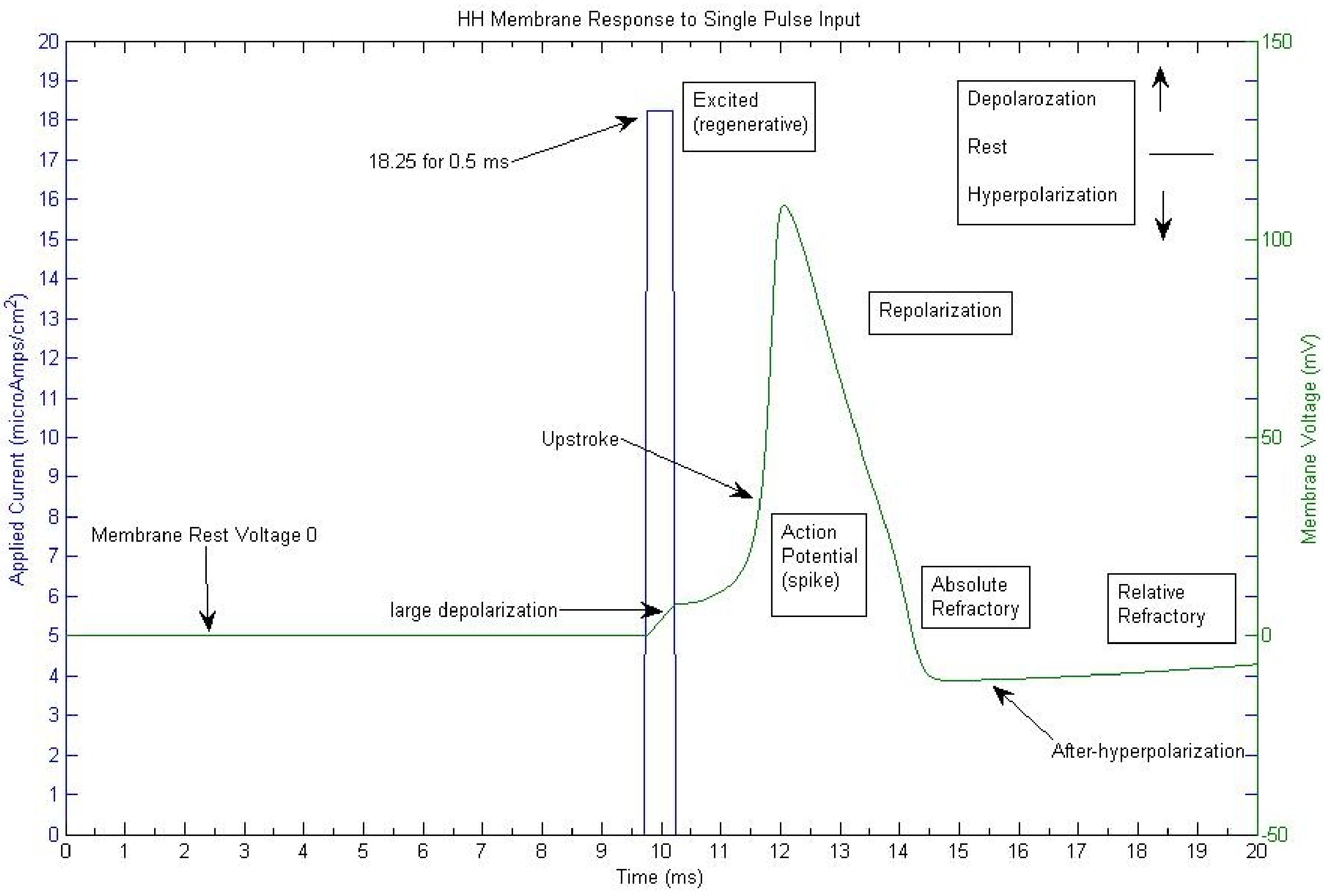

Simulink Block Diagram of I_Na m producer (Level 0)
Initial Condition: m=0.0529324852572496

# I_L leak (Level 1)

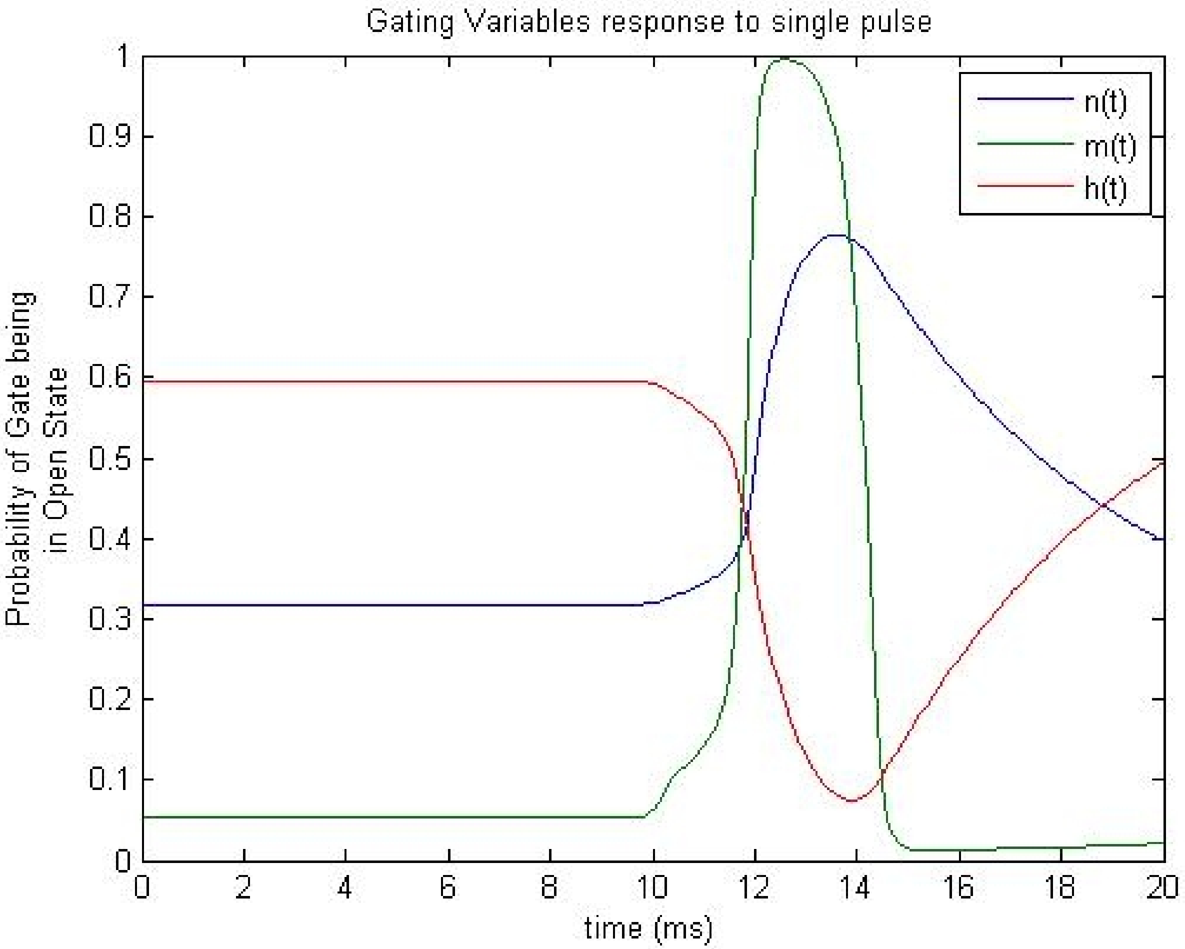



Simulink Block Diagram of I_Leak (Level 1)

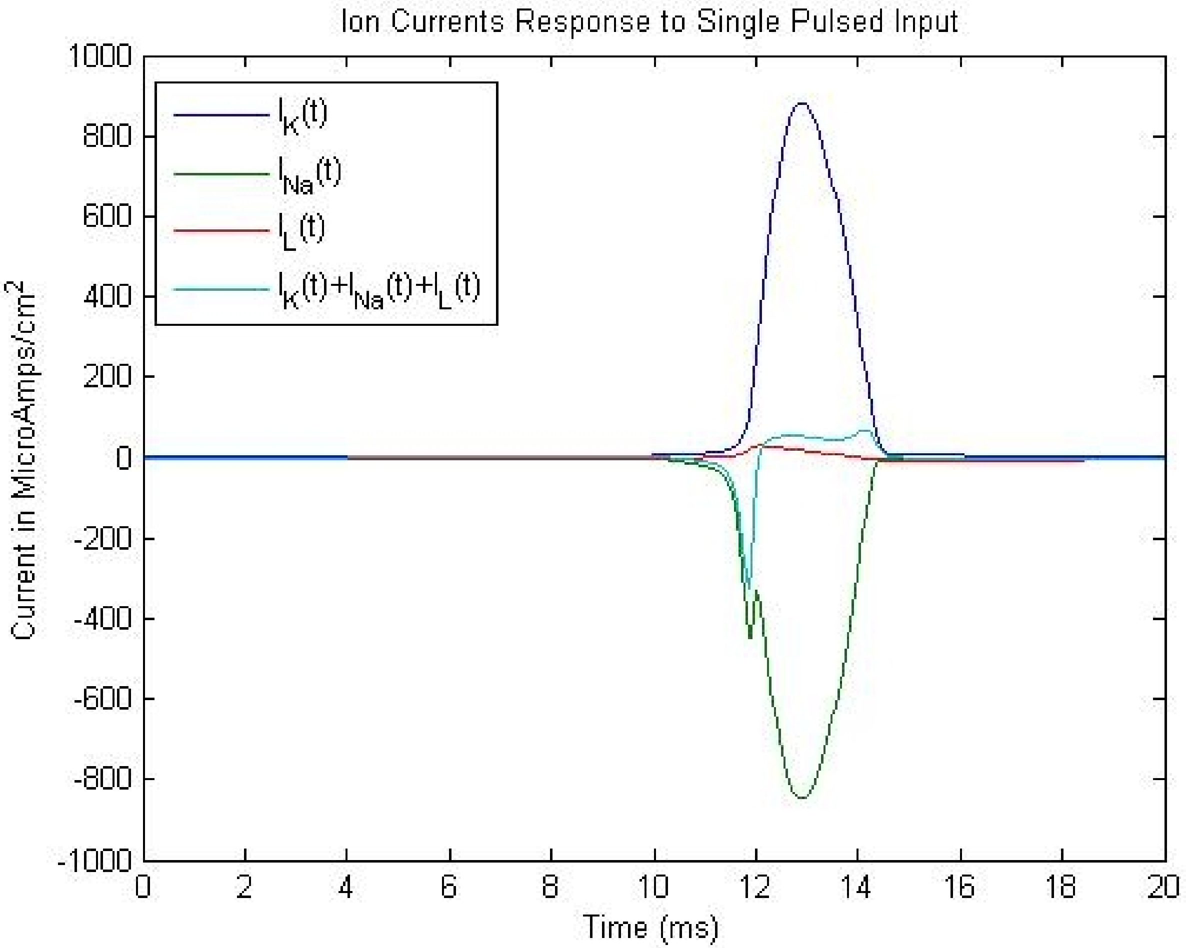
# HH Spike Event Membrane Voltage



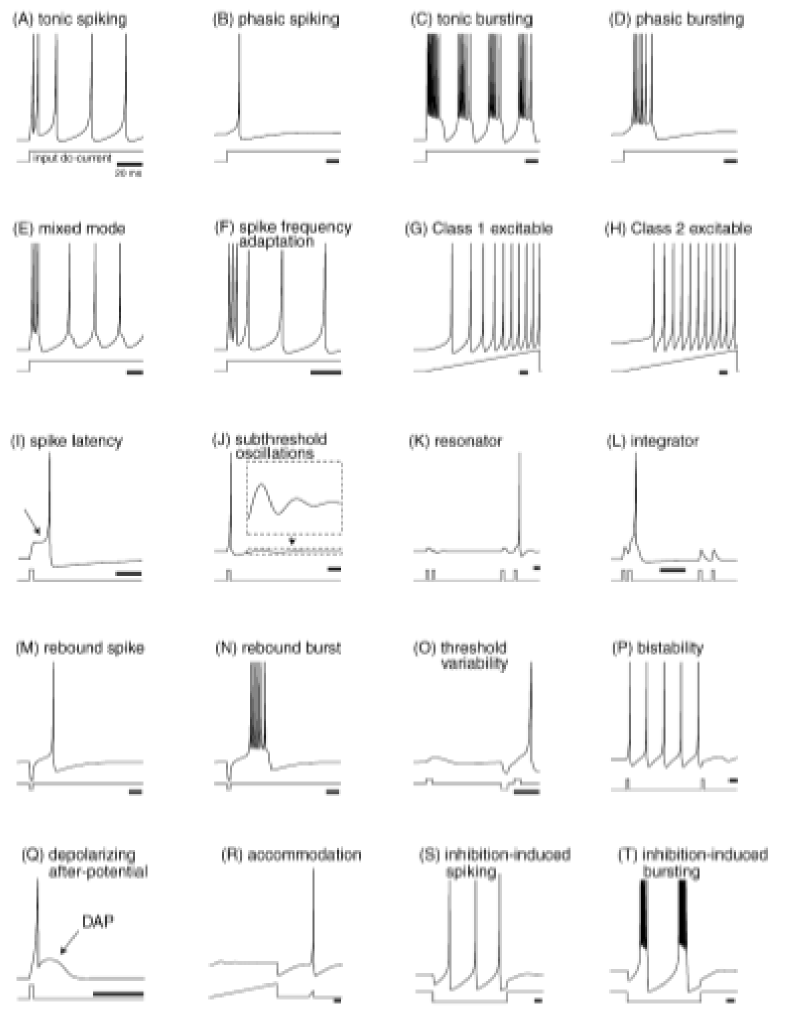
# HH Spike Event Gating Variables



# HH Spike Event
# Ion Currents



# HH Dynamical Responses




Izhikevich, Eugene M. Which Model to Use for Cortical Spiking Neurons? IEEE TRANS. ON NEURAL NETS, VOL. 15, NO. 5 [2004]

# Dynamical Responses of Spiking Neuron Models

| Models | biophysically meaningful | tonic spiking | phasic spiking | tonic bursting | phasic bursting | mixed mode | spike frequency adaptation | class 1 excitable | class 2 excitable | spike latency | subthreshold oscillations | resonator | integrator | rebound spike | rebound burst | threshold variability | bistability | DAP | accommodation | inhibition-induced spiking | inhibition-induced bursting | chaos | # of FLOPS |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| integrate-and-fire | - | + | - | - | - | - | - | + | - | - | - | - | + | - | - | - | - | - | - | - | - | - | 5 |
| integrate-and-fire with adapt. | - | + | - | - | - | - | + | + | - | - | - | - | + | - | - | - | - | + | - | - | - | - | 10 |
| integrate-and-fire-or-burst | - | + | + | | + | - | + | + | - | - | - | - | + | + | + | - | + | + | - | - | - | | 13 |
| resonate-and-fire | - | + | + | - | - | - | - | + | + | - | + | + | + | + | - | - | + | + | + | - | - | + | 10 |
| quadratic integrate-and-fire | - | + | - | - | - | - | - | + | - | + | - | - | + | - | - | + | + | - | - | - | - | - | 7 |
| Izhikevich (2003) | - | + | + | + | + | + | + | + | + | + | + | + | + | + | + | + | + | + | + | + | + | + | 13 |
| FitzHugh-Nagumo | - | + | + | - | | - | - | + | - | + | + | + | - | + | - | + | + | - | + | + | - | - | 72 |
| Hindmarsh-Rose | - | + | + | + | | | + | + | + | + | + | + | + | + | + | + | + | + | + | + | | + | 120 |
| Morris-Lecar | + | + | + | - | | - | - | + | + | + | + | + | + | + | | + | + | - | + | + | - | - | 600 |
| Wilson | - | + | + | + | | | + | + | + | + | + | + | + | + | + | + | | + | + | | | | 180 |
| Hodgkin-Huxley | + | + | + | + | | | + | + | + | + | + | + | + | + | + | + | + | + | + | + | | + | 1200 |

Izhikevich, Eugene M. Which Model to Use for Cortical Spiking Neurons? IEEE TRANS. ON NEURAL NETS, VOL. 15, NO. 5 [2004]